

# Kolmogorov Dissipation scales in Weakly Ionized Plasmas

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## ABSTRACT

In a weakly ionized plasma, the evolution of the magnetic field is described by a “generalized Ohm’s law” that includes the Hall effect and the ambipolar diffusion terms. These terms introduce additional spatial and time scales which play a decisive role in the cascading and the dissipation mechanisms in magnetohydrodynamic turbulence. We determine the Kolmogorov dissipation scales for the viscous, the resistive and the ambipolar dissipation mechanisms. The plasma, depending on its properties and the energy injection rate, may preferentially select one of these dissipation scales, thus determining the shortest spatial scale of the supposedly self-similar spectral distribution of the magnetic field. The results are illustrated taking the partially ionized part of the solar atmosphere as an example. Thus the shortest spatial scale of the supposedly self-similar spectral distribution of the solar magnetic field is determined by any of the four dissipation scales given by the viscosity, the Spitzer resistivity (electron-ion collisions), the resistivity due to electron-neutral collisions and the ambipolar diffusivity. It is found that the ambipolar diffusion dominates for reasonably large energy injection rate. The robustness of the magnetic helicity in the partially ionized solar atmosphere would facilitate the formation of self-organized vortical structures.

**Key words:** Partially ionized plasma, Kolmogorov dissipation, Hall effect, Ambipolar diffusion.

## 1 INTRODUCTION

It is widely recognized that some ideal integrals of motion (i.e., constants of motion in the dissipation-less limit) play an essential role in the complex nonlinear dynamics leading to remarkable self-organizing structures. The “helicity” is one such quantity that imposes a “topological” constraint on a field Moffat (1978). Constancy of the helicity is not attributed to any “symmetry” of the system, but is due to a “defect” (or a singularity) of the Poisson-bracket operator, and, thus, is a robust constraint throughout the evolution.

A dissipative mechanism can change the helicity to remove the topological constraint. Self-organization may be understood as a subtle balance between the conservation (restriction) and the dissipation (relaxation).

One of the most successful models of self-organization is due to Taylor (1974), who invoked the constancy of the magnetic helicity  $H = \int \mathbf{A} \cdot \mathbf{B} d^3x/2$  (the integral is taken over the total volume). Minimizing the magnetic energy  $E = \int |\mathbf{B}|^2 d^3x/2$  (he omitted the kinetic and the thermal energies) for a fixed  $H$  yields an Euler-Lagrange equation  $\nabla \times \mathbf{B} = \lambda \mathbf{B}$  ( $\lambda$  is the Lagrange multiplier for restricting  $H$ ), whose solution is the “Beltrami field” representing a twisted force-free magnetic field. The cross helicity  $H_c = \int \mathbf{V} \cdot \mathbf{B} d^3x/2$  is also an invariant of the ideal magnetohydrodynamic system with flows where  $\mathbf{V}$  is the fluid velocity.

This “variational principle” is based on the assumption that the energy  $E$  is preferentially (selectively) dissipated while the helicity  $H$  is approximately conserved (and that no other conserved quantity puts an obstacle for the minimization of  $E$ ). A possible justification for this assumption is given by the “energy cascade” ansatz —the energy density of the fluctuating field cascades toward small spatial scales, and the variations (spatial derivatives) of the field are enhanced. Then, the dissipation of the energy, which includes higher-order spatial derivatives in comparison with the helicity, proceeds much faster than that of the helicity. This “scale change” of fluctuations may be represented by the cascade of the energy density in the Fourier (wave number) space.

It has been established that  $E$  and  $H$  are the integrals of motion of an ideal magnetohydrodynamic system, the energy cascade yields the selective dissipation of the energy (with respect to the helicity) leading to the Taylor relaxed state. This paper extends the scope of these considerations to “weakly ionized plasmas”. There are many astrophysical systems with a rather low degree of ionization dominated by the charged particle-neutral collisions

and the neutral particle dynamics. A major part of the solar photosphere (Leake and Arber 2006; Krishan and Varghese 2008;), the protoplanetary disks (Krishan and Yoshida 2006) and the molecular clouds (Brandenburg and Zweibel 1994) are some of the examples of weakly ionized astrophysical plasmas. In addition these systems are believed to be turbulent. The evolution of the magnetic fields in such a plasma would be affected by the multifluid interactions in general and the ambipolar diffusion in particular (Zweibel 1988). It is important to know which dissipation scale would be the most effective in a given situation.

Here, We (1) set up the energy and the helicity evolution equations in a weakly ionized plasma and (2) determine the Kolmogorov dissipation scales for the different energy dissipation mechanisms. The weakly ionized part of the solar atmosphere is presented as an example to illustrate the predominance of one or the other dissipation scale. We show that the ambipolar effect, which is nonlinearly enhanced when the energy injection rate is large, may dominate the energy dissipation, while it conserves the helicity.

## 2 ENERGY AND HELICITIES IN WEAKLY IONIZED PLASMAS

The dynamics of a weakly ionized plasma can be described with the following equations (Krishan and Yoshida 2006, Krishan and Varghese 2008, Krishan and Gangadhara, 2008): the momentum balance of the neutral fluid is given by:

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = -\nabla h + \frac{\mathbf{J} \times \mathbf{B}}{c\rho_n} + \mu \nabla^2 \mathbf{V}, \quad (1)$$

and the magnetic field  $\mathbf{B}$  evolves as:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left\{ \left[ \mathbf{V} - \frac{\mathbf{J}}{en_e} + \frac{\mathbf{J} \times \mathbf{B}}{c\nu_{in}\rho_i} \right] \times \mathbf{B} \right\} + \eta \nabla^2 \mathbf{B}. \quad (2)$$

and

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} \quad (3)$$

where  $\mathbf{V}$  is the velocity of the neutral fluid and  $h$  is the total enthalpy. The Lorentz force in the neutral dynamics appears due to the ion- neutral coupling in that the Lorentz force on the ions ( $en_i(\mathbf{E} + \frac{\mathbf{V}_i \times \mathbf{B}}{c})$ ) is balanced by the ion-neutral collisional force ( $-\rho_i \nu_{in}(\mathbf{V}_i - \mathbf{V}_n)$ ) where  $\nu_{in}$  is the ion- neutral collision frequency. The ion inertial force has been neglected in comparison with the ion-neutral frictional force. The viscosity  $\mu$  of the neutral fluid arises due to neutral-neutral collisions with frequency  $\nu_{nn}$ . The electron inertial force is also neglected. We have further assumed a constant density incompressible system. We may write the electric diffusivity  $\eta$  as:

$$\eta = \frac{c^2}{4\pi} \frac{m_e \nu_{en}}{n_e e^2} = \delta_e^2 \nu_{en}, \quad (4)$$

and the kinematic viscosity  $\mu$  as:

$$\mu = \frac{v_{n,th}^2}{\nu_{nn}} = \lambda_n^2 \nu_{nn}, \quad (5)$$

where  $\delta_e := c/\omega_{pe} = c/\sqrt{4\pi n_e e^2/m_e}$  is the electron inertial length, and  $\lambda_n := v_{n,th}/\nu_{nn}$  is the neutral mean free path:  $\nu_{en}$ ,  $\nu_{nn}$  and  $\nu_{ei}$  are the electron-neutral, the neutral-neutral, and the electron-ion collision frequencies;  $\nu_{ei} \ll \nu_{en}$  in the case of the weakly ionized plasma.

Assuming constant densities and incompressibility, we normalize the variables in the ‘‘Alfvénic units’’. We chose a representative neutral flow velocity  $V_0$  and a magnetic field  $B_0$  related as:

$$\frac{\rho_n V_0^2}{2} = \frac{B_0^2}{8\pi}. \quad (6)$$

Solving (6) for  $V_0$  gives a virtual Alfvénic velocity  $V_0 = B_0/(4\pi\rho_n)^{1/2}$ . If  $\rho_n$  were  $\rho_i$ , this  $V_0$  is the well-known Alfvén velocity. In a weakly ionized plasma,

$$\alpha := \frac{\rho_n}{\rho_i} \gg 1 \quad (7)$$

For example, in the solar atmosphere,  $\alpha$  can be as large as  $10^6$ , the protoplanetary disks have  $\alpha$  of the order of  $10^8$ .

Let  $L_0$  be a characteristic length in the system. We normalize  $\mathbf{x}$  by  $L_0$ ,  $t$  by  $t_0 := L_0/V_0$ ,  $\mathbf{B}$  by  $B_0$ ,  $\mathbf{V}$  by  $V_0$  and energy densities by  $\rho_n V_0^2 = B_0^2/(8\pi)$ . Using these variables, Equations (1) and (2) can be written in the (1) and (2) dimension-less form:

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = -\nabla h + \mathbf{J} \times \mathbf{B} + \epsilon_\mu \nabla^2 \mathbf{V}, \quad (8)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \{[\mathbf{V} - \epsilon_H \mathbf{J} + \epsilon_A \mathbf{J} \times \mathbf{B}] \times \mathbf{B}\} + \epsilon_\eta \nabla^2 \mathbf{B}, \quad (9)$$

where the scaling parameters are defined as:

$$\epsilon_H := \alpha \frac{c/\omega_{pi}}{L_0} = \alpha \frac{\delta_i}{L_0}, \quad (10)$$

$$\epsilon_A := \epsilon_H \frac{\omega_{ci}}{\nu_{in}}, \quad (11)$$

$$\epsilon_\eta := \eta \frac{t_0}{L_0^2} = \eta \frac{1}{L_0 V_0}, \quad (12)$$

$$\epsilon_\mu := \mu \frac{t_0}{L_0^2} = \mu \frac{1}{L_0 V_0}, \quad (13)$$

Here,  $\delta_i$  is the ion inertial length. The scaling parameter  $\epsilon_H$  multiplying the Hall term is enhanced by the factor  $\alpha = \rho_n/\rho_i$  in comparison with the standard (fully ionized) Hall term.

With an appropriate homogeneous boundary conditions, we have the energy equation ( $E = \int (V^2 + B^2) dx/2$ ),

$$\frac{dE}{dt} = -\epsilon_A \int J_{\perp}^2 B^2 d^3x - \epsilon_{\eta} \int J^2 d^3x - \epsilon_{\mu} \int |\nabla \times \mathbf{V}|^2 d^3x, \quad (14)$$

and, for completeness, the helicity equation ( $H = \int \mathbf{A} \cdot \mathbf{B} dx/2$ )

$$\frac{dH}{dt} = -\epsilon_{\eta} \int \mathbf{J} \cdot \mathbf{B} d^3x. \quad (15)$$

In the ideal limit ( $\epsilon_A = 0, \epsilon_{\eta} = 0$  and  $\epsilon_{\mu} = 0$ ),  $E$  and  $H$  are conserved. The constancy of  $H$  is destroyed only by a finite resistivity  $\epsilon_{\eta}$ . The cross helicity  $H_c$  of the ideal MHD transforms to the ion canonical helicity  $H_G = \int (\mathbf{A} + \epsilon_H \mathbf{V}) \cdot \nabla \times (\mathbf{A} + \epsilon_H \mathbf{V}) dx/2$ . The cross helicity  $H_c$  is conserved when  $\epsilon_H = 0, \epsilon_A = 0$  and the ion helicity  $H_G$  is conserved for  $\epsilon_A = 0$ . For  $\epsilon_A \neq 0$  both  $H_c$  and  $H_G$  are not conserved. From (14), we find that the energy dissipation is contributed by (i) ambipolar diffusion (scaled by  $\epsilon_A$ ), (ii) resistivity (friction of electrons with neutrals; scaled by  $\epsilon_{\eta}$ ), and (iii) neutral viscosity (scaled by  $\epsilon_{\mu}$ ). The advective terms  $(\mathbf{V} \cdot \nabla) \mathbf{V}$ ,  $\nabla \times (\mathbf{V} \times \mathbf{B})$  and  $\nabla \times (\mathbf{J} \times \mathbf{B})$  ( the Hall term ) are responsible for the energy cascade mechanism. Since the velocity and the magnetic field are coupled through equations (8) and (9), all the advective and the dissipative processes operate on both the velocity and the magnetic fields. We shall define the Reynolds number in a rather broad sense as the ratio of the advective term and the dissipation term. We will see that the Reynolds numbers resulting from different combinations of the advective process and the dissipation process set up a rather complex scale hierarchy of the Kolmogorov microscale, in the wave number ( $k$ ) space, which changes depending on the strength of the energy injection rate. In what follows we will select one dissipation mechanism and combine it with the three advective ( cascading ) mechanisms to define the three Reynolds numbers and determine the corresponding Kolmogorov microscales.

### 3 SCALE HIERARCHY

The wave-number ( $k$ ) space is divided into (i) the energy injection (large scale) range, (ii) the “inertial range”, and (iii) the dissipation range. The inertial range is dominated by the convective  $[(\mathbf{V} \cdot \nabla) \mathbf{V}]$ , inductive  $[\nabla \times (\mathbf{V} \times \mathbf{B})]$  and the Hall  $[-\nabla \times (\epsilon_H \mathbf{J} \times \mathbf{B})]$  effects, which create a sub-hierarchy in the inertial range. The higher- $k$ -end of the inertial range is the “Kolmogorov scale” that is determined by one of the three (ambipolar, viscous and

resistive) dissipation mechanisms. The aim of this section is to estimate the Kolmogorov scale by identifying the responsible (i.e., the dominant) mechanism of energy dissipation.

Here we invoke Kolmogorov's ansatz of "local interactions" in the  $k$ -space, which may be formulated as follows. Let  $K$  symbolize a "range" of wave-number space, i.e.,  $K$  stands for  $\{\mathbf{k}; K \leq |\mathbf{k}| < K + \Delta\}$ , where  $\mathbf{k}$  is the wave vector and  $\Delta$  is a certain positive constant. Denoting by  $\hat{u}(\mathbf{k})$  the Fourier transform of a field  $u(\mathbf{x})$ , we define  $u_K = \sum_{\mathbf{k} \in K} \hat{u}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{x}}$ , which means the component of  $u(\mathbf{x})$  in the hierarchy of wave-numbers ranging in  $K$ . Suppose a term  $X$  is included in an evolution equation of a physical quantity  $u$ . When we observe the hierarchy  $K$ ,  $X_K$  contributes the temporal variation  $(\partial u / \partial t)_K$ . If  $X$  is a linear term including a field  $v$  (and, possibly, the differential operator  $\nabla$ ), then we may estimate  $X_K$  only by  $v_K$ . But, if  $X$  includes, for example,  $v \cdot w$ , then all  $\hat{v}(\mathbf{k}_1)$  and  $\hat{w}(\mathbf{k}_2)$  (and the corresponding  $\nabla$  translating into  $i\mathbf{k}_1$  and  $i\mathbf{k}_2$ ) may contribute to  $X_K$ , if  $\mathbf{k}_1 \pm \mathbf{k}_2 \in K$ . Now, the "locality ansatz" claims that only  $v_K$  and  $w_K$  (and, thus,  $\nabla$  of order  $K$ ) dominates  $X_K$ . We assume that this ansatz holds for all (even higher-order) nonlinear terms in our system. We also assume that no geometric anisotropy diminishes the magnitudes of vector products such as  $\mathbf{J} \times \mathbf{B}$ . Hence, we estimate, for example,  $|\mathbf{J} \times \mathbf{B}|_K \approx |\mathbf{J}|_K |\mathbf{B}|_K \approx K |\mathbf{B}|_K^2$  (using  $\mathbf{J} = \nabla \times \mathbf{B}$  in the normalized unit). In what follows, we denote  $|\mathbf{B}|_K$  by  $B_K$ .

### 3.1 Kolmogorov scales defined by viscosity dissipation

Using a rather general definition of the Reynolds number as the ratio of the term responsible for the cascade process to that responsible for the dissipation process, we consider, first, the advective term  $(\mathbf{V} \cdot \nabla) \mathbf{V}$  and the dissipation due to the viscosity. This defines the very familiar, the (standard) Kolmogorov scale, when the other mechanisms (ambipolar diffusion and resistivity) are still negligible.

The Reynolds number is defined by

$$R_\mu(K) := \frac{|(\mathbf{V} \cdot \nabla) \mathbf{V}|_K}{|\epsilon_\mu \nabla^2 \mathbf{V}|_K}. \quad (16)$$

Note that this Reynolds number is evaluated for each scale hierarchy as a function of  $K$ . Invoking the previous assumptions, we estimate

$$R_\mu(K) \approx \frac{V_K}{\epsilon_\mu K}. \quad (17)$$

The "Kolmogorov microscale" is usually defined as the spatial scale  $l$  at which the Reynolds number becomes unity ( e.g. Holmes, Lumley and Berkosz 1996, Frisch 1995 ). We prefer to work in the wavevector space such that  $K = l^{-1}$  and define the inverse Kolmogorov

microscale  $K = K_\mu$  where  $R_\mu(K)$  becomes of the order of unity ( e.g. Pope 2000). We obtain

$$K_\mu = \frac{V_{K_\mu}}{\epsilon_\mu}. \quad (18)$$

To estimate (eliminate)  $V_{K_\mu}$ , we invoke the energy cascade rate ( total energy  $E$  being an invariant of the system)  $\mathcal{E}$  ( normalized by  $\mathcal{E}_0 = V_0^3/L_0$  ) that is assumed to be scale invariant and equal to the energy dissipation rate as well as the energy injection rate. From Eq.(13)

$$\mathcal{E} = \epsilon_\mu K_\mu^2 V_{K_\mu}^2 = \frac{V_{K_\mu}^4}{\epsilon_\mu}. \quad (19)$$

Solving (19) for  $V_{K_\mu}$ , we obtain

$$V_{K_\mu} = \mathcal{E}^{1/4} \epsilon_\mu^{1/4}. \quad (20)$$

Substituting (20) into (18) yields the well-known relation

$$K_\mu = \mathcal{E}^{1/4} \epsilon_\mu^{-3/4}. \quad (21)$$

If the energy cascade is dominated by the advective term  $\nabla \times (\mathbf{V} \times \mathbf{B})$  and the dissipation by the viscosity; the Reynolds number  $R_{\mu M}(K)$  should better represent the relation between the energy cascade and the dissipation where

$$R_{\mu M}(K) := \frac{|\nabla \times (\mathbf{V} \times \mathbf{B})|_K}{\epsilon_\mu |\nabla^2 \mathbf{V}|_K} \approx \frac{B_K}{\epsilon_\mu K} = \frac{V_K}{\epsilon_\mu K C_{V/B}}, \quad (22)$$

where we have introduced a coefficient  $C_{V/B} := V_K/B_K$  that is, in general, a function of  $K$ .

The corresponding Kolmogorov scale becomes

$$K_{\mu M} = \frac{B_{K_{\mu M}}}{\epsilon_\mu} = \frac{V_{K_{\mu M}}}{\epsilon_\mu C_{V/B}} = \mathcal{E}^{1/4} \epsilon_\mu^{-3/4} C_{V/B}^{-1}. \quad (23)$$

In the ideal MHD regime, however, we may assume  $C_{V/B} \approx 1$  throughout the MHD inertial range, because the energy transfer is an equal collaboration of the induction and the convection terms. Hence,  $K_{\mu M} \approx K_\mu$ .

The situation changes, when the Hall term dominates the energy cascade. This is the case if the ion skin depth multiplied by the density ratio  $\alpha$ ,  $\epsilon_H$  (normalized by the system size) is larger than the Kolmogorov length scale, i.e.,  $\epsilon_H K_\mu > 1$ . One can define the Hall Reynolds number by taking the advective term  $\epsilon_H \mathbf{J} \times \mathbf{B}$  ( the Hall term) and the dissipative term to be due to the viscosity. The corresponding Reynolds number is defined as

$$R_{\mu H}(K) := \frac{|\epsilon_H \nabla \times (\mathbf{J} \times \mathbf{B})|_K}{\epsilon_\mu |\nabla^2 \mathbf{V}|_K} \approx \frac{\epsilon_H B_K^2}{\epsilon_\mu V_K} = \frac{\epsilon_H V_K}{\epsilon_\mu C_{V/B}^2} \quad (24)$$

scales the ratio of the energy cascade rate and the dissipation. Assuming  $C_{V/B} = (\epsilon_H K)^p$  (for  $K > 1/\epsilon_H$ ) with a certain exponent  $p$  (the Hall-MHD turbulence theory, Krishan and

Mahajan 2005, predicts, for R-mode turbulence  $p = 1$ , while for L-mode turbulence  $p = -1$ ), we obtain a ‘‘Hall Kolmogorov scale’’ (denoting  $q := 1/(2p)$ )

$$K_{\mu H} = \epsilon_H^{q-1} \left( \frac{V_{K_{\mu H}}}{\epsilon_\mu} \right)^q = \epsilon_H^{q-1} \left( \mathcal{E}^{1/4} \epsilon_\mu^{-3/4} \right)^q = \left( \epsilon_H^{-1} \right)^{1-q} (K_\mu)^q. \quad (25)$$

Since  $K_{\mu H}/K_\mu = 1/(\epsilon_H K_\mu)^{1-q}$  (and we are assuming  $\epsilon_H K_\mu > 1$ ),  $K_{\mu H}$  interpolates  $\epsilon_H^{-1}$  and  $K_\mu$  (i.e.,  $\epsilon_H^{-1} < K_{\mu H} < K_\mu$ ) as long as  $q < 1$ .

As mentioned above, the condition for the appearance of the Hall MHD regime is

$$\epsilon_H K_\mu = \epsilon_H \epsilon_\mu^{-3/4} \mathcal{E}^{1/4} > 1. \quad (26)$$

### 3.2 Kolmogorov scales defined by resistivity dissipation

Next, we examine the case where the resistive dissipation dominates over the viscous and the ambipolar diffusivities. Both the viscous and the resistive dissipations have a common mathematical structure viz. they are linear terms multiplied by  $\nabla^2$ .

The resistive and the viscous dissipation mechanisms may be compared as follows: from (14), the ratio of the resistive and viscous dissipation terms is

$$\frac{|\epsilon_\eta \nabla \times \mathbf{B}|_K^2}{|\epsilon_\mu \nabla \times \mathbf{V}|_K^2} = \frac{\epsilon_\eta K^2 B_K^2}{\epsilon_\mu K^2 V_K^2} = \frac{\epsilon_\eta}{\epsilon_\mu} C_{V/B}^{-2}. \quad (27)$$

Again taking  $(\mathbf{V} \cdot \nabla) \mathbf{V}$  as the advective term, in the MHD regime where we may assume  $C_{V/B} \approx 1$ , the resistive Kolmogorov scale is given by just replacing  $\epsilon_\mu$  by  $\epsilon_\eta$  in (21), i.e.,

$$K_\eta = \mathcal{E}^{1/4} \epsilon_\eta^{-3/4}, \quad (28)$$

where the energy dissipation rate is  $\mathcal{E} = \epsilon_\eta K_\eta^2 B_{K_\eta}^2$ . Evaluating  $\mathcal{E}$  for the same energy injection rate, we may write  $K_\eta = (\epsilon_\eta/\epsilon_\mu)^{-3/4} K_\mu$ . By (4) and (5), we estimate

$$\frac{\epsilon_\eta}{\epsilon_\mu} = \frac{\delta_e^2 \nu_{en}}{\lambda_n^2 \nu_{nn}}. \quad (29)$$

For the MHD regime where we may assume  $C_{V/B} \approx 1$  the advective term  $\nabla \times (\mathbf{V} \times \mathbf{B})$  along with the resistive dissipation furnishes the same scale  $K_\eta$ .

In the Hall MHD regime ( $\epsilon_H K_\eta > 1$ ), we replace the previous Hall Reynolds number (24) by

$$R_{\eta H}(K) := \frac{|\epsilon_H \nabla \times (\mathbf{J} \times \mathbf{B})|_K}{|\epsilon_\eta \nabla^2 \mathbf{B}|_K} \approx \frac{\epsilon_H B_K}{\epsilon_\eta}. \quad (30)$$

Hence, at the corresponding Kolmogorov scale  $K_{\eta H}$ , we estimate  $B_{K_{\eta H}} = \epsilon_\eta/\epsilon_H$ . From the energy dissipation rate  $\mathcal{E} = \epsilon_\eta K_{\eta H}^2 B_{K_{\eta H}}^2$ , we obtain

$$K_{\eta H} = \mathcal{E}^{1/2} \epsilon_\eta^{-3/2} \epsilon_H = K_\eta^2 \epsilon_H. \quad (31)$$



Hence, unlike the relation of the viscosity Kolmogorov scales (25),  $K_{\eta H} > K_\eta$ .

### 3.3 Kolmogorov scales defined by ambipolar diffusion

Finally, we estimate the Kolmogorov scales assuming that the dissipation is dominated by the ambipolar term.

We begin with the MHD regime where we may assume  $C_{V/B} \approx 1$ . As shown earlier in this case the advective terms  $(\mathbf{V} \cdot \nabla)\mathbf{V}$  and  $\nabla \times (\mathbf{V} \times \mathbf{B})$  furnish equal Kolmogorov microscales. We define an “ambipolar Reynolds number” by

$$R_A(K) := \frac{|\nabla \times (\mathbf{V} \times \mathbf{B})|_K}{|\epsilon_A \nabla \times [(\mathbf{J} \times \mathbf{B}) \times \mathbf{B}]|_K} \approx \frac{V_K}{\epsilon_A K B_K^2}. \quad (32)$$

The ambipolar Kolmogorov scale  $K_A$  is characterized by  $R_A(K_A) \approx 1$ :

$$K_A = \frac{V_{K_A}}{\epsilon_A B_{K_A}^2} = \frac{C_{V/B}}{\epsilon_A B_{K_A}}. \quad (33)$$

The ambipolar dissipation rate is now (Eq. 13):

$$\mathcal{E} = \epsilon_A K_A^2 B_{K_A}^4 = \epsilon_A^{-1} C_{V/B}^2 B_{K_A}^2. \quad (34)$$

Plugging  $B_{K_A} = \mathcal{E}^{1/2} \epsilon_A^{-1/2} C_{V/B}^{-1}$  into (33), and assuming  $C_{V/B} \approx 1$ , we obtain

$$K_A = \epsilon_A^{-3/2} \mathcal{E}^{-1/2}. \quad (35)$$

Let us compare (35) with (21):

$$\frac{K_A}{K_\mu} = \epsilon_\mu^{3/4} \epsilon_A^{-3/2} \mathcal{E}^{-3/4}. \quad (36)$$

From this relation, we see that the ambipolar diffusion dominates over the viscous dissipation (i.e.,  $K_A/K_\mu \ll 1$ ), when the energy dissipation rate (=energy injection rate)  $\mathcal{E}$  is sufficiently large. This is because the ambipolar dissipation is a nonlinear term that is enhanced when the fluctuation level is high.

If the Hall term dominates the energy cascade i.e. taking  $\epsilon_H \nabla \times (\mathbf{J} \times \mathbf{B})$  as the advective term ( $\epsilon_H K_A > 1$ ), we replace (32) by

$$R_{AH}(K) := \frac{|\epsilon_H \nabla \times (\mathbf{J} \times \mathbf{B})|_K}{|\epsilon_A \nabla \times [(\mathbf{J} \times \mathbf{B}) \times \mathbf{B}]|_K} \approx \frac{\epsilon_H}{\epsilon_A B_K}. \quad (37)$$

Using (34), we estimate  $B_{K_{AH}} = (\mathcal{E}/\epsilon_A)^{1/4} K_{AH}^{-1/2}$ , which, together with  $R_{AH}(K_{AH}) = 1$  yields

$$K_{AH} = \epsilon_A^{3/2} \epsilon_H^{-2} \mathcal{E}^{1/2} = (\epsilon_H K_A)^{-2} K_A. \quad (38)$$

We, thus, see  $K_{AH} < K_A$ .

Comparing with (25), we get:

dissipation mechanism	MHD regime ( $\epsilon_H K < 1$ )	H MHD regime ( $\epsilon_H K > 1$ )
viscosity	$K_\mu = \mathcal{E}^{1/4} \epsilon_\mu^{-3/4}$	$K_{\mu H} = (\epsilon_H^{-1})^{1-q} K_\mu^q \quad (< K_\mu)$
resistivity	$K_\eta = \mathcal{E}^{1/4} \epsilon_\eta^{-3/4}$	$K_{\eta H} = \epsilon_H K_\eta^2 \quad (> K_\eta)$
ambipolar	$K_A = \mathcal{E}^{-1/2} \epsilon_A^{-3/2}$	$K_{AH} = \epsilon_H^{-2} K_A^{-1} \quad (< K_A)$

**Table 1.** Summary of Kolmogorov scales

$$\frac{K_{AH}}{K_{\mu H}} = \epsilon_A^{3/2} \epsilon_H^{-(q+1)} \epsilon_\mu^{3q/4} \mathcal{E}^{(2-q)/4} = \left( \frac{\omega_{ci}}{\nu_{in}} \right)^{3/2} \epsilon_H^{(1/2)-q} \epsilon_\mu^{3q/4} \mathcal{E}^{(2-q)/4}. \quad (39)$$

### 3.4 Comparison of different Kolmogorov scales

In Table 1, we summarize different estimates of Kolmogorov scales. When the plasma parameters and the energy injection rate  $\mathcal{E}$  are specified, we can estimate the appropriate Kolmogorov scale determined by the balance between the relevant energy cascade and the dissipation mechanisms.

## 4 SCALE HIERARCHY ON THE SOLAR ATMOSPHERE

The solar magnetic flux, believed to be generated in the convection zone, has to pass through the partially ionized solar photosphere before it can appear high up in the solar corona. This realization is rather recent and is now receiving a lot of attention. Arber, Haynes and Leake (2007) has emphasized the profound effects on the temperature and the current structure of the overlying chromosphere and the corona that the inclusion of the neutral medium can produce. It is important to know which dissipation scale would be the most effective for the conditions typical of the solar atmosphere. We estimate the various Kolmogorov scales in the partially ionized part of the solar atmosphere.

The various collision frequencies are determined from Khodochenko et al.(2004):

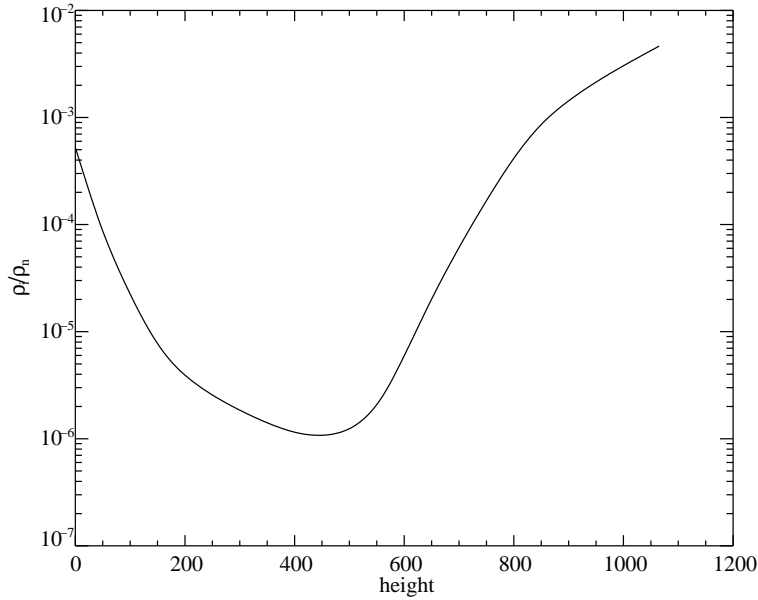
$$\nu_{ij} = \Sigma_{ij} n_n \left( \frac{8K_B T}{\pi m_{ij}} \right)^{1/2}, \quad (40)$$

where (i,j) stands for the species of particles, the cross-section  $\Sigma_{en} \approx 10^{-15} \text{cm}^2$ ,  $\Sigma_{in} \approx \Sigma_{nn} \approx 5 \times 10^{-15} \text{cm}^2$ ,  $m_{ij} = m_i m_j (m_i + m_j)^{-1}$ ,  $n_n$  is the neutral particle density and  $T$  is temperature in degree Kelvin. The typical values of the physical parameters on the solar atmosphere are given in Table II.

The variation of the ionization fraction  $\frac{\rho_i}{\rho_n}$  with height on the solar atmosphere is shown in figure (1).

The ratios of the different Kolmogorov scales can be expressed as:

$h$	$T$	$\rho_i$	$\rho_n$	$B$	$B_e$
0.	6520.	$1.00 \times 10^{-10}$	$1.90 \times 10^{-07}$	1200.00	$9.4 \times 10^{-04}$
50.	5790.	$1.20 \times 10^{-11}$	$1.59 \times 10^{-07}$	1125.77	$1.2 \times 10^{-04}$
125.	5270.	$1.18 \times 10^{-12}$	$1.00 \times 10^{-07}$	980.16	$1.3 \times 10^{-05}$
175.	5060.	$3.39 \times 10^{-13}$	$7.04 \times 10^{-08}$	880.33	$4.6 \times 10^{-06}$
250.	4880.	$9.37 \times 10^{-14}$	$3.89 \times 10^{-08}$	737.21	$1.7 \times 10^{-06}$
400.	4560.	$1.12 \times 10^{-14}$	$1.09 \times 10^{-08}$	503.71	$4.2 \times 10^{-07}$
490.	4410.	$4.37 \times 10^{-15}$	$4.84 \times 10^{-09}$	394.42	$2.6 \times 10^{-07}$
560.	4430.	$4.72 \times 10^{-15}$	$2.47 \times 10^{-09}$	322.27	$4.2 \times 10^{-07}$
650.	4750.	$2.29 \times 10^{-14}$	$1.00 \times 10^{-09}$	246.31	$3.7 \times 10^{-06}$
775.	5280.	$1.08 \times 10^{-13}$	$3.79 \times 10^{-10}$	183.67	$3.5 \times 10^{-05}$
855.	5650.	$1.75 \times 10^{-13}$	$1.66 \times 10^{-10}$	143.40	$1.0 \times 10^{-04}$
980.	5900.	$1.78 \times 10^{-13}$	$6.57 \times 10^{-11}$	108.65	$1.8 \times 10^{-04}$
1065.	6040.	$1.67 \times 10^{-13}$	$3.60 \times 10^{-11}$	90.88	$2.5 \times 10^{-04}$



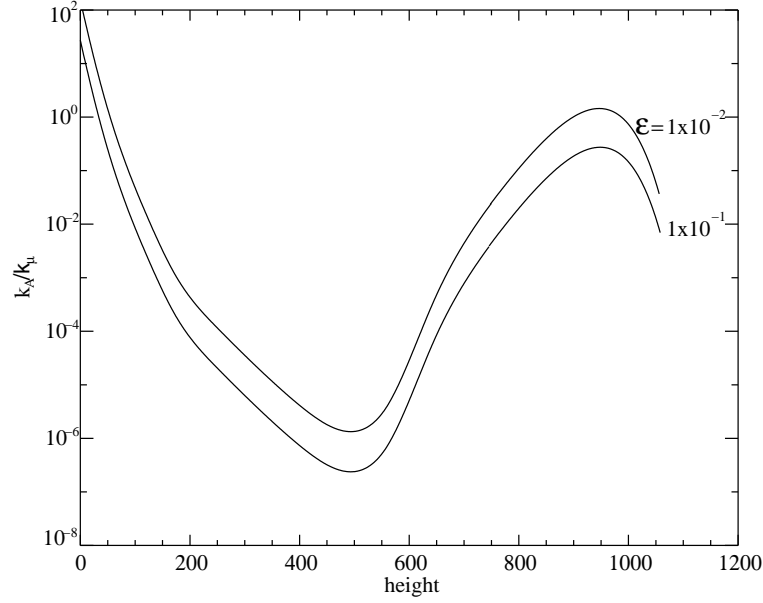
**Figure 1.** Variation of the Ionization fraction  $\frac{\rho_i}{\rho_n}$  with height  $h$  on the solar atmosphere.

$$\frac{K_A}{K_\mu} = 3 \times 10^{18} \mathcal{E}^{-3/4} \rho_i^{9/4} \rho_n^{-3/4} T^{9/8} B^{-3/2}, \quad (41)$$

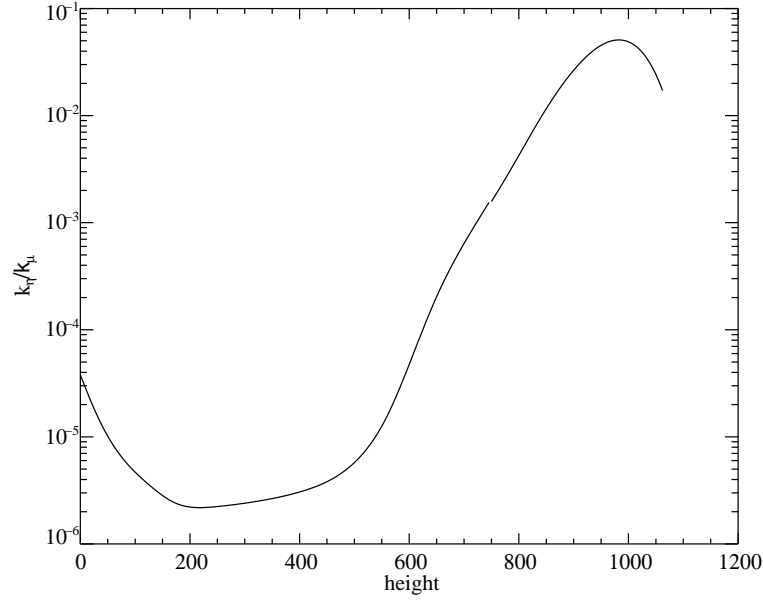
$$\frac{K_\eta}{K_\mu} = 10^{-7} \rho_i^{3/4} \rho_n^{-3/2} \quad (42)$$

We present plots of the ratios  $\frac{K_A}{K_\mu}$ ,  $\frac{K_\eta}{K_\mu}$  and  $\frac{K_A}{K_\eta}$  vs height for different values of the energy injection rate  $\mathcal{E}$  vs height in Figs. (2), (3) and (4) respectively.

We observe from Fig.(2) that the ambipolar dissipation predominates over the viscous dissipation i.e.  $K_A < K_\mu$  on major part of the solar photosphere and the chromosphere. Choosing the parameters at a height of 980 Km above the photosphere as the normalizing parameters i.e.  $\rho_n = 6.5 \times 10^{-11} \text{ g cm}^{-3}$ ,  $B = 108 \text{ G}$ ,  $L_0 \approx 1000 \text{ Km}$ , we find  $V_0 \approx 3.5 \text{ Km s}^{-1}$  and  $\mathcal{E}_r = 4 \times 10^8 \text{ cm}^2 \text{ s}^{-3}$ . The condition for the predominance of the ambipolar diffusion over the viscous dissipation becomes  $\mathcal{E} > 1.2 \times 10^{-2}$  or, recovering dimensions,  $5 \times 10^6 \text{ cm}^2 \text{ s}^{-3}$ .

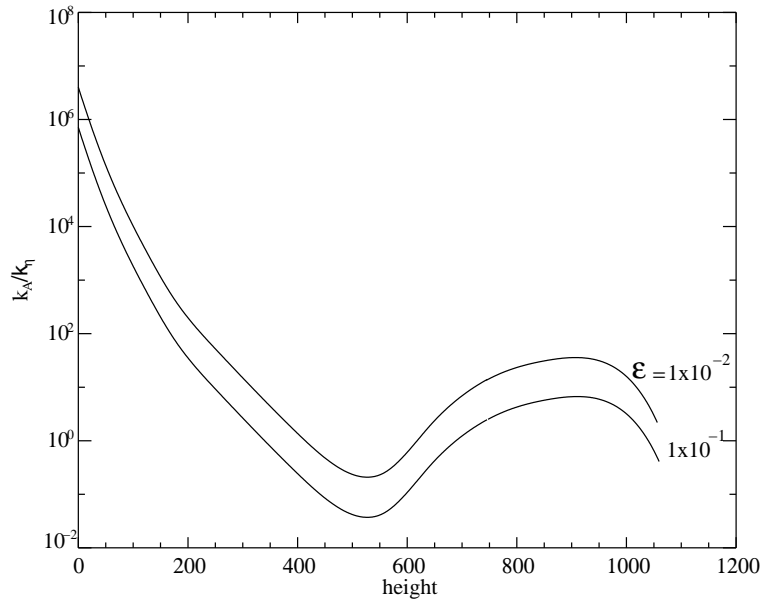


**Figure 2.** Variation of the ratio  $\frac{K_A}{K_\mu}$  with height  $h$  for different values of the energy injection rate  $\mathcal{E}$



**Figure 3.** Variation of the ratio  $\frac{K_\eta}{K_\mu}$  with height  $h$

One can estimate the typical injection rate of the turbulent convective energy on the sun to be  $V_c^3/L_0 \approx 10^8 \text{ cm}^2 \text{ s}^{-3}$  by taking a typical convective velocity  $V_c \approx 2 \text{ km s}^{-1}$ . The condition for the predominance of the ambipolar diffusion is, thus, easily satisfied. The inclusion of the Hall effect modifies  $K_A$  to  $K_{AH}$  and as seen from table I  $K_{AH} < K_A$ , reconfirming the predominance of the ambipolar effect as the dissipation mechanism. Figure (3) demonstrates the predominance of the resistive dissipation (due to electron-neutral collisions) over the viscous dissipation.



**Figure 4.** Variation of the ratio  $\frac{K_A}{K_\eta}$  with height  $h$  for different values of the energy injection rate  $\mathcal{E}$

That the ambipolar dissipation predominates over the resistive dissipation i.e.  $K_A < K_\eta$  for the region extending from a height of 200 Km to 1000 Km for reasonable values of the energy injection rate  $\mathcal{E}$  can be seen in figure (4). The Spitzer resistivity in a weakly ionized plasma, by definition, is much smaller than the resistivity due to the electron-neutral collisions. We conclude that the ambipolar dissipation would dominate over other dissipation mechanisms on most of the partially ionized region of the solar atmosphere.

## 5 CONCLUSION

The dynamics of weakly ionized plasmas is governed by the neutral fluid being subjected to the Lorentz force and the magnetic induction to the Hall and and the ambipolar effects. There are now three mechanisms by which the energy could dissipate, each mechanism having its own characteristic dissipation scale. We have determined these scales as presented in Table I. The example of the solar atmosphere shows that the ambipolar dissipation would determine the short scale end of the supposedly self-similar distribution of the photospheric magnetic field thereby identifying the region of dissipation leading to heating. We observe that  $H$  is a more robust quantity in comparison with  $E$ , if either the ambipolar or the viscous dissipation is larger than the resistive dissipation. In the standard argument of selective dissipation, moreover, even the resistivity dissipates  $E$  faster than it does  $H$  because of the energy cascade toward small scales;  $dE/dt$  includes higher-order spatial derivatives in

comparison with  $dH/dt$ , so that  $dE/dt$  assumes a larger value for small-scale fields. This could lead to the formation of organized plasma structures, vortical structures with twisted magnetic fields, so ubiquitous on the sun.

## ACKNOWLEDGMENTS

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